Engineering Notes

Engine-Airframe Coupling in Liquid Rocket Systems

K. J. McKenna,* J. H. Walker,* and R. A. Winje*
TRW Space Technology Laboratories, Redondo Beach, Calif.

Nomenclature

A	= propellant line cross-sectional area
A_t	= nozzle throat area
B, C_a	= spring constant and damping associated with accumulator
C	= nozzle characteristic velocity
C_f	= thrust coefficient
g	= acceleration due to gravity
h	= propellant height in tank
H	= head rise across pump
K	= const
L	= propellant inertance
M	= structure generalized mass
ϕ	= modal displacement
P	= pressure
R	= propellant line resistance
S	= Laplace operator
T	= thrust
V	= vapor volume
$\dot{W}$	= propellant flow rate
X	= displacement at thrust point
γ	= propellant density
ζ	= fraction of critical damping
τ	= thrust chamber time constant
ω	= structural frequency

Subscripts

a	= accumulator
c	= thrust chamber
d	= discharge side of pump
f	= fuel
g	= thrust point
o	= oxidizer
p	= pump
s	= suction side of pump
t	= tank bottom

LOW-FREQUENCY longitudinal vibration has been observed on several large liquid rocket boosters. These have included the Thor-Agena, Atlas-Agena, Titan I, and Titan II. The vibration occurs during the latter portion of boost flight and has a frequency corresponding to the frequency of the fundamental longitudinal mode of the vehicle. Fundamental mode frequencies for the forementioned vehicles range between 10 and 30 cps. Oscillations at these frequencies are also observed in propellant feed system pressures and in thrust chamber pressure.

The vibration results from limit cycling of a closed-loop system that is formed by coupling between the flexible airframe and the booster engine. The mechanism involved may be described by assuming a disturbance in the thrust acting on the vehicle. This disturbance excites the vibratory modes of the structure, thereby producing oscillations in propellant supply pressures. These oscillating pressures cause variations in the propellant flow rate into the combustion chamber and, consequently, cause variations in thrust that can reinforce the original disturbance and cause the oscillations to diverge. The amplitudes of oscillation are then limited by nonlinearities in the system.

Vibration amplitudes can be reached that may produce large axial loads in the vehicle structure, create a severe vibration environment for delicate payloads, or degrade propulsion system performance. The mechanism of the phenomenon is such that it should be considered as a potential problem on all liquid rocket systems. It is the purpose of this note to present a simple linear mathematical model that can be used to examine the longitudinal stability of an arbitrary liquid rocket booster. The application of the model to an analysis of the Titan II longitudinal vibration problem and the methods used to eliminate the problem[1, 2] are briefly discussed. Other work in the general field of engine-airframe coupling is given in Refs. 3–5.

Model Development

In formulating the linear mathematical model that follows, a typical vehicle was separated into a number of elements, and equations of motion were written to describe the dynamic characteristics of the elements and their interactions. The origin of the coordinate system is located at the center of mass of the vehicle. The equations are written to give deviation of the variables from their steady-state values; hence, steady-state terms have been eliminated.

Structure

Even though the vehicle structure is quite complex, modal characteristics for the lower modes can be calculated accurately by considering the vehicle as a series of concentrated masses and springs. Propellants in the tanks are included in the structural model, but propellants in the feed lines are considered separately. Techniques for constructing this

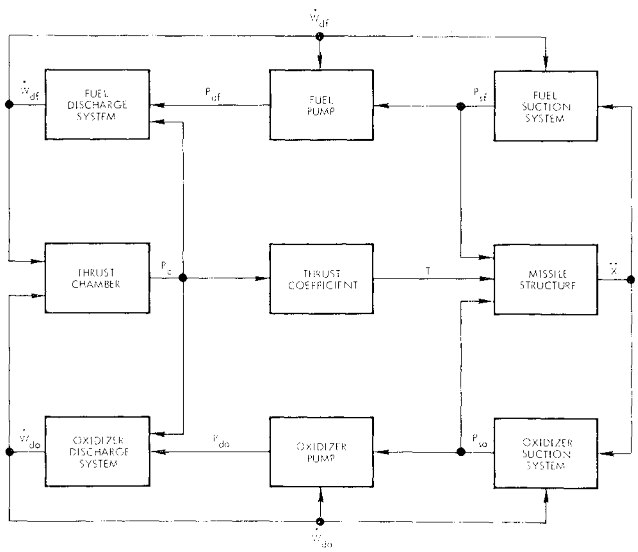

Fig. 1 Longitudinal model block diagram.

Presented as Preprint 64-81 at the AIAA Aerospace Sciences Meeting, New York, January 20–22, 1964; revision received October 19, 1964. This work was supported by Ballistic Systems Division, Air Force Systems Command, Contract No. AF 04(694)-1.

* Member of the Technical Staff.

type of structural model are given in Ref. 7. Particular attention should be paid to the mass-spring representation of such components as the propellant pumps and the thrust chambers, since the coupling between the airframe and engine occurs at these points.

Based on the vibration frequencies that have been observed on current vehicles, it is probable that only the fundamental structural mode will be of interest. The first mode stuctural response in generalized coordinates is given by Eq. (1) where modal frequencies and amplitudes are calculated from the mass-spring model:

$$(S^2 + 2\zeta\omega S + \omega^2)X = \frac{\phi_g^2}{M} T - A_s \frac{\phi_g\phi_p}{M} P_s \tag{1}$$

On the right-hand side of Eq. (1) are the generalized force due to thrust and the generalized force that results when the propellant in the feed system is coupled to the structure at the point p.

Propulsion system

The propulsion system consists of the propellant in the tanks, the suction systems, propellant pumps, discharge systems, and thrust chambers. The relation of tank bottom pressure to structural acceleration is given by Eq. (2):

$$P_t = (\gamma h/g)(\phi_t/\phi_g)S^2X \tag{2}$$

Titan II flight data shows that tank ullage pressure does not oscillate during the vibration period; thus this term does not appear in the equation. However, studies on the Atlas vehicle[6] indicate a coupled structural/pneumatic system longitudinal instability immediately after lift-off when the tank ullage volume is comparatively small and this pressure is important.

The propellant suction systems are considered as separate dynamic systems for two reasons. First, small vapor bubbles can exist at the pump inlet due to pump cavitation.[5] Also, extensive dynamic tests on the Titan II pump-suction system[2] revealed the existence of a characteristic frequency associated with the pump-suction system that was dependent on the pump cavitation index. The compliance in the suction system is made up of contributions from conduit elasticity, fluid compressibility, and vapor compressibility. Since no theoretical means have been found for estimating this compliance, the suction system is considered separately and the compliance is lumped as a vapor bubble of volume V at the pump inlet. The suction pressure is given by Eq. (3) and the vapor volume is given by (4), which is a linearized perfect gas law:

$$P_s = P_t - L_s S \dot{W}_s + L_s S \dot{W}_a \tag{3}$$

$$V = -P_s/K \tag{4}$$

Since the suction system was considered separately, it is coupled to the structure through the use of a generalized force as given in Eq. (1).

Continuity of fluid flow through the pump, including the vapor bubble described previously, is given by

$$\dot{W}_s = \dot{W}_d - \gamma A(\phi_p/\phi_g)SX - \gamma SV \tag{5}$$

If changes in pump rotative speed are neglected, the pressure rise across the pumps is given by

$$P_d = \left(\frac{\partial H}{\partial P_s} + 1\right) P_s + \frac{\partial H}{\partial \dot{W}_d} \dot{W}_d \tag{6}$$

Tests performed on the Titan II pumps[2] indicate that for the frequencies of interest (0–30 cps) the values of the partial derivatives in Eq. (6) can be obtained by taking the slopes of steady-state pump performance curves.

Discharge systems of pump fed engines usually involve short pipes and high pressures, so that compliance in the system need not be considered. Therefore, considering only

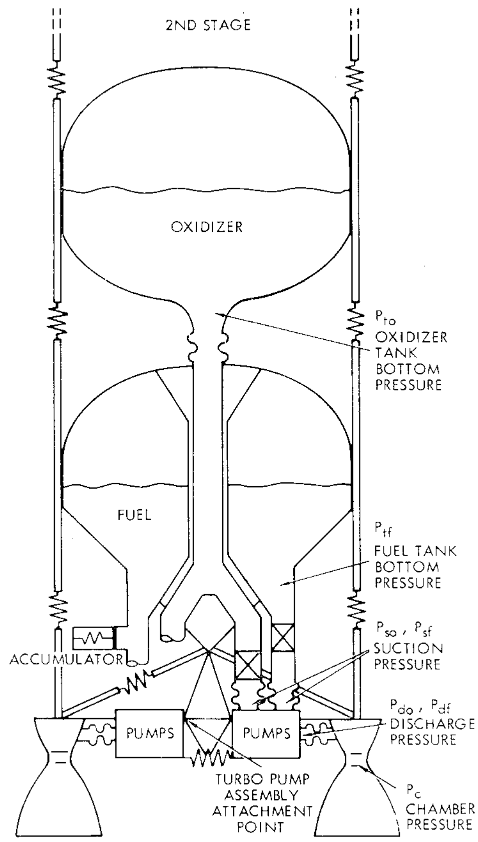

Fig. 2 Engine-airframe schematic for first stage of Titan II.

inertance of the fluid in the line and resistance (including injector resistance), the equation for the discharge system is

$$(SL_d + R_d)\dot{W}_d = P_d - P_c \tag{7}$$

Equations (8) and (9) are assumed to be a valid representation of the thrust chamber for low frequencies and small perturbations:

$$(\tau S + 1)P_c = C\dot{W}_d/A_t g \tag{8}$$

$$T = A_t C_f P_c \tag{9}$$

Equations (1–9) comprise the mathematical model. The linearized equations are amenable to standard frequency response and stability analyses techniques. A block diagram showing the interaction between the various model elements of a bipropellant vehicle is shown in Fig. 1.

Application

A schematic of the Titan II Stage I is shown in Fig. 2. The Titan II is a bipropellant missile with two booster engines, each fed by separate propellant lines. Linear system equations were written as described previously and are represented in the functional block diagram (Fig. 1). Once the constant coefficients in the system equations were determined, frequency response and stability analysis techniques were applied. These coefficients are readily calculable or available from system design data and engine performance data. However, it was only after extensive testing of the Titan II pump-suction system that characteristic suction system frequencies were known. The coupling between the propulsion system and the structure is highly dependent on the proximity of the suction system frequencies to the structural frequency. The limit cycle amplitude, as observed in flight, cannot be duplicated by the linear model. However, the analysis does give the correct onset and cessation of vibration and gives information on the relative importance of various elements to system stability.

To eliminate the vibration, hydraulic accumulators were placed in the propellant suction lines near the pump inlets. This is shown schematically in the left fuel line in Fig. 2. The accumulator is a spring mass device that alters the dynamic characteristics of the suction line. By proper choice of accumulator constants, the propulsion system can effectively be decoupled from the structure. Mathematically, the accumulator is included in the model through the equation

$$(L_aS^2 + C_aS + B)W_a = -P_s \qquad (10)$$

and the addition of the last term in Eq. (3). To date, accumulators have been incorporated in four Titan II flights and have essentially eliminated the vibration phenomenon.

References

[1] Walker, J. H., Winje, R. A., and McKenna, K. J., "An investigation of low frequency longitudinal vibration of the Titan II missile during stage I flight," TRW/Space Technology Labs., Rept. 6438-6001-RU000, Contract AF 04(694)-479 (March 1964)

[2] Walker, J. H. and Winje, R. A., "An investigation of low frequency longitudinal vibration of the Titan II missile during stage I flight—Addendum," TRW/Space Technology Labs., Rept. 6438-6001-RU001, Contract AF 04(694)-479 (June 1964).

[3] Wick, R. S., "The effect of vehicle structure on combustion stability in liquid propellant rockets," Jet Propulsion Labs., Pasadena, Calif., Progr. Rept. 20-248, Sherman M. Fairchild Fund Paper FF-34 (December 1954).

[4] Majoros, J. and Sarlat, I. M., "Propulsion perturbation effect on missile dynamics," IAS 31st Annual Meeting (January 1963).

[5] McDonald, D. and Calvert, T. R., "Design considerations of large space vehicles due to axial oscillations caused by engine-structural coupling," *Shock, Vibration and Associated Environments*, Office of the Secretary of Defense, Washington, D.C., Part IV, Bull. 33 (March 1964).

[6] Rose, R. G. and Harris, R., "Dynamic analysis of a coupled structural pneumatic system longitudinal oscillation for Atlas vehicles," AIAA Paper 64-483 (July 1964); also J. Spacecraft Rockets (submitted for publication).

[7] Wood, J. D., "Survey on missile structural dynamics," Space Technology Labs., Redondo Beach, Calif., Rept. 7102-0041-NU000, Contract AF 04(647)-619 (June 1961).

Ballistics of Solid Propellants during Thrust Modulation

B. Dubrow,* E. D. Guth,† and M. W. Wong†

TRW Space Technology Laboratories, Redondo Beach, Calif.

Nomenclature

a = reaction constant from the Summerfield burning rate relationship
A_b = area of propellant burning
A_t = area of the throat, thrust modulating nozzle
A_{t_0} = area of the throat, initial
b = diffusion constant from the Summerfield burning rate relationship
C_W = mass flow factor
M = molecular weight
P = pressure
P_0 = initial pressure

Presented as Preprint 64-130 at the AIAA Solid Propellant Rocket Conference, Palo Alto, Calif., January 29–31, 1964; revision received November 2, 1964.

* Member of the Technical Staff. Member AIAA.

† Member of the Technical Staff.

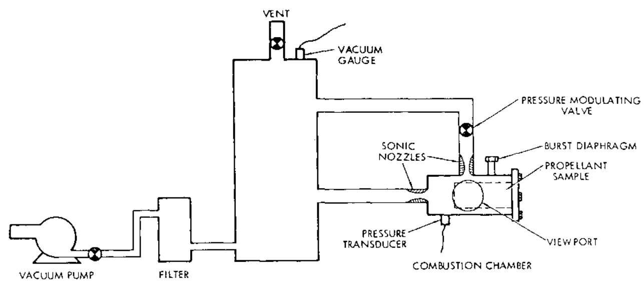

Fig. 1 Schematic diagram of experimental apparatus.

R = gas constant
r = linear burning rate of solid propellant
T = absolute temperature
t = time required for the pressure to change from P_0 to P
V = volume
ρ = propellant density
ρ_g = gas density

Introduction

THRUST modulation of conventional solid propellant rockets can be made to occur through controlled pressure excursions in the rocket chamber. The chamber pressure, in turn, can be varied by changes in the propellant burning surface area, the propellant properties, or the nozzle throat area. Changing the nozzle throat area is considered as the method that lends itself most readily to reducing the concept of thrust modulation to practice. In order to determine the feasibility, problems, and design criteria related to thrust modulation of solid propellants by programed variations of the nozzle throat area, the solid propellant ballistics must be characterized under transient conditions of pressure rise and decay.

The reported experimental investigations of pressure decay did not appear to be devised for general analytical treatment. Furthermore, the efforts were aimed at generating data under the limiting conditions in the region of combustion extinction. The analysis of programed thrust modulation requires more inclusive experimental conditions. In the interest of arriving at a general treatment of thrust modulation, an experimental program was devised which could generate data required for analytical assessment of the related ballistics.

Analytical Treatment of Transient Ballistics

The internal ballistics for a solid rocket can be expressed analytically in general form by the mass balance equation:

$$Vd\rho_g/dt = \rho A_b r - C_W P A_t \qquad (1)$$

If the propellant continues to burn during thrust modulation, introduction of an expression of the burning rate as a function of pressure permits Eq. (1) to be solved for time as a function of pressure. The Summerfield equation for pressure-dependence of burning rate is a two-term parametric formula:

$$1/r = a/P + b/P^{1/3} \qquad (2)$$

Equation (2) considers the combustion mode for solid propellants as consisting of two rate-controlling phenomena. At low pressure, chemical reaction is rate controlling, whereas diffusion is the predominant rate controlling process at high pressure.

Equation (1) can be solved when Eq. (2) is used to describe the burning rate, yielding

$$t = \frac{MV}{RT}\left\{\frac{a}{A_b\rho - aC_W(A_{t_0} + A_t)}\ln\frac{P}{P_0} + \frac{3}{2}\left[\frac{C_W(A_{t_0} + A_t)}{A_b\rho - aC_W(A_{t_0} + A_t)}\right] \times \ln\left[\frac{[A_b\rho/C_W(A_{t_0} + A_t)] - (a + bP^{2/3})}{[A_b\rho/C_W(A_{t_0} + A_t)] - (a + bP^{2/3})}\right]\right\} \qquad (3)$$